BOUNDARY CONDITION FOR THE EQUATION OF PARTICLE DIFFUSION IN NONUNIFORM FLOW

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The boundary condition on an absorbing surface for the equation of particle diffusion is obtained from the approximate solution of the Fokker–Planck equation.

One of the most complex problems associated with an analysis of finely dispersed flows is the calculation of particle deposition on surface exposed to the flow. Calculations for fine-particle deposition are usually accomplished on the basis of a diffusion equation (the Smoluchowski equation). This leads to a problem with the formulation of the boundary conditions at those surfaces on which the particles settle out. Particle deposition can also be calculated by solving an equation for the probability density of particle transition from one point in space to another (the Kolmogorov equation [1]). The boundary conditions for the equation of transition probability were obtained in [1, 2]; whence it follow, in particular, that the density of transition probability on a totally absorbing wall is equal to zero. However, the mechanical transfer of the boundary conditions for the transition-probability density to the particle-concentration distribution is incorrect, since the concentration is determined by averaging the transition-probability density over all possible particle trajectories. Therefore, particle concentration on a totally absorbing wall, unlike the transition-probability density, may not be equal to zero, as has been demonstrated in a numbered of references, for example [3, 4], by solution of the Fokker–Planck equation of particles.

In solving the diffusion equations, we most often assume the particle concentration in the vicinity of an absorbing surface to be equal to zero, which is not always valid. On occasion, the particle concentration n_W at a surface is assumed as given, which is a rather artificial approach, since n_W (as well as the particle flow J_W through a wall) is the sought quantity and may change along the surface exposed to the flow. It is proposed in [5] to assign boundary conditions of the third kind $\partial n/\partial y \sim (n - n_W)/\sqrt{\tau D}$ at some distance $y_0 \sim \sqrt{\tau D}$ from the wall, with D denoting the Brownian diffusion coefficient. A condition similar in form is derived in [6] from a model based on the equality between the diffusion particle flow from an external region and the inertial flow $D(\partial n/\partial y) = V_{y1} n/2\alpha$ to a wall over the inertial run y_1, where D is the coefficient of Brownian and turbulent diffusion; V_{y1} is the mean particle velocity in the inertial region at the wall, α is that number of particles remaining on the wall when the flow comes into contact with the wall. The boundary-value problem for the particle-diffusion equation is also discussed in [7-10] as well as in other works. In this article, the boundary condition for the diffusion equation is obtained from the Fokker–Planck kinetic equation for particles.

Particle motion in a nonuniform random field can be described by the Fokker–Planck equation for the distribution function in phase space f(t, x, v). The Fokker–Planck equation can be used to describe both the Brownian diffusion [3-5] and particle motion in the field of carrier-phase turbulent pulsations [11]. Let us write this equation in the form

$$\frac{\partial f}{\partial t} + v_k \frac{\partial f}{\partial x_k} + \left(\frac{U_k - V_k}{\tau} - F_k\right) \frac{\partial f}{\partial v_k} = \frac{1}{\tau} \frac{\partial (v_k - V_k) f}{\partial v_k} + \frac{D}{\tau^2} \frac{\partial^2 f}{\partial v_k \partial v_k}. \quad (1)$$

where the terms proportional to $1/\tau$ describe the force of the interphase interaction in the Stokes approximation, while F_k is the external force acting on the particle (for example, the force of gravity). The diffusion coefficient D in Eq. (1) is assumed to be dependent on x, i.e., consideration is given to the spatial nonuniformity of the pulsation field.

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Having integrated Eq. (1) over the entire volume in velocity space, we obtain the equation of continuity for particle concentration

$$\frac{\partial n}{\partial t}+\frac{\partial nV_k}{\partial x_k}=0, \tag{2}$$

where the concentration and mean velocity of the particles are determined from the relationships

$$n=\int f d\mathbf{v}, \quad \mathbf{V}=\frac{1}{n}\int \mathbf{v} f d\mathbf{v}.$$

Having multiplied Eq. (1) by V_i and integrating in the velocity space, we obtain the equation for particle motion

$$\frac{\partial V_i}{\partial t}+V_k\frac{\partial V_i}{\partial x_k}=-\frac{1}{n}\frac{\partial (P_{ik}n)}{\partial x_k}+\frac{U_i-V_i}{\tau}+F_i, \tag{3}$$

where the stress tensor P_{ik} is defined by the relationship

$$P_{ik}=\frac{1}{n}\int (V_i-v_i)(V_k-v_k) f dv.$$

Assuming only a slight deviation of the system from local equilibrium, where in the first approximation we can neglect the left-hand side of Eq. (1), the distribution function f is described by the Maxwell distribution

$$f_0=\left(\frac{\tau}{2\pi D}\right)^{3/2} n \exp\left(-\frac{\tau|\mathbf{v}-\mathbf{V}^2|}{2D}\right), \tag{4}$$

which satisfies the right-hand side of Eq. (1).

Based on Eq. (4), $P_{ik} = D\delta_{ik}/\tau$, and Eq. (4) assumes the form of

$$\frac{\partial V_i}{\partial t}+V_k\frac{\partial V_i}{\partial x_k}=-\frac{1}{n\tau}\frac{\partial (Dn)}{\partial x_i}+\frac{U_i-V_i}{\tau}+F_i. \tag{5}$$

With D = const, Eq. (5) coincides with the equation for the Brownian motion of aerosol particles [12]. The term containing the gradient $\partial D/\partial x_i$ describes the effect of particle migration in a nonuniform random flow [9, 13].

In the diffusion approximation $\tau D/\ell^2 \ll 1$ it follows from Eq. (5) that

$$V_i=U_i+\tau F_i-\frac{1}{n}\frac{\partial (Dn)}{\partial x_i}. \tag{6}$$

With consideration of Eq. (6), the continuity equation (2) assumes the form of the diffusion equation

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x_k}[(U_k+\tau F_k)n]=\frac{\partial^2 (Dn)}{\partial x_k \partial x_k}. \tag{7}$$

To obtain the boundary condition for Eq. (7), let us construct a solution for Eq. (1) in the kinetic layer near the surface. We can retain those terms in Eq. (1) that apply to this layer and relate to the direction y of the projection onto the normal to the wall:

$$\frac{D}{\tau}\frac{\partial^2 f}{\partial v_y^2}+\frac{\partial v_y f}{\partial v_y}=\tau v_y\frac{\partial f}{\partial y}+(U_y+\tau F_y)\frac{\partial f}{\partial v_y}. \tag{8}$$

Equation (8) is solved by the method of perturbations, where in order to obtain the boundary condition for Eq. (7), in addition to the first (Maxwell) approximation, it is necessary also to construct a second approximation. As the small parameter for this problem we can take the quantity $\varepsilon = \sqrt{\tau D}/\ell$ equal to the ratio of the kinetic-layer thickness $\sqrt{\tau D}$ to the characteristic dimension of the region of variation for the average parameters in the direction ℓ across the surface. Thus, the solution to Eq. (3) is presented in the form

$$f = f_0 + f_1 + \dots = \left(\frac{\tau}{2\pi D}\right)^{1/2} n \exp\left(-\frac{\tau v_y^2}{2D}\right) + f_1 + \dots,$$

where the function f_1 is determined from the following equation:

$$\frac{D}{\tau}\frac{\partial^2 f_1}{\partial v_y^2} + \frac{\partial v_y f_1}{\partial v_y} = \tau v_y \frac{\partial f_0}{\partial y} + (U_y + \tau F_y)\frac{\partial f_0}{\partial v_y} =$$
$$= \left(\frac{\tau^3}{2\pi D}\right)^{1/2} v_y \exp\left(-\frac{\tau v_y^2}{2D}\right)\left[\frac{dn}{dy} + \frac{n}{2D}\left(\frac{\tau v_y^2}{D} - 1\right)\frac{dD}{dy} - (U_y + \tau F_y)\frac{n}{D}\right]. \tag{9}$$

A solution for Eq. (9) is given by

$$f_1 = -\left(\frac{\tau^3}{2\pi D}\right)^{1/2} v_y \exp\left(-\frac{\tau v_y^2}{2D}\right)\left[\frac{dn}{dy} + \frac{n}{2D}\left(1 + \frac{\tau}{3D}v_y^2\right)\frac{dD}{dy} - (U_y + \tau F_y)\frac{n}{D}\right].$$

Thus, the solution for Eq. (8) in the second approximation is of the form

$$f = \left(\frac{\tau}{2\pi D}\right)^{1/2} n \exp\left(-\frac{\tau v_y^2}{2D}\right)\left\{1 - \frac{\tau}{D} v_y\left[D\frac{d\ln n}{dy} + \left(\frac{1}{2} + \frac{\tau}{6D}v_y^2\right)\frac{dD}{dy} - (U_y + \tau F_y)\right]\right\}. \tag{10}$$

On the basis of solution (10), let us determine the particle flows J_{inc} incident on the wall and J_{ref} reflected from the wall:

$$J_{inc} = -\int_{-\infty}^{0} v_y f dv_y = \left(\frac{D}{2\pi\tau}\right)^{1/2} n + \frac{1}{2}\frac{dDn}{dy} - (U_y + \tau F_y)\frac{n}{2}; \tag{11}$$

$$J_{ref} = \int_{0}^{\infty} v_y f dv_y = \left(\frac{D}{2\pi\tau}\right)^{1/2} n - \frac{1}{2}\frac{dDn}{dy} + (U_y + \tau F_y)\frac{n}{2}. \tag{12}$$

All of the quantities in Eqs. (11) and (12) correspond to their values at the wall ($y = 0$). We will characterize the surface properties by the reflection coefficient χ which is equal to the probability that a particle, having reached the wall, will break away from the wall (i.e., the return into the flow of a particle having collided with the wall); here $\alpha = 1 - \chi$ is the probability that the particle will adhere to the wall (i.e., to leave the flow). Consequently, the coefficient χ is equal to the ratio of the particle flows reflected from and incident on the surface:

$$\chi = J_{ref}/J_{inc}. \tag{13}$$

Having substituted (11) and (12) into (13), we obtain the following boundary condition for the particle-diffusion equation:

$$(1-\chi) n_w = (1+\chi)\left(\frac{\pi\tau}{2D_w}\right)^{1/2}\left[D_w\left(\frac{dn}{dy}\right)_w + n_w\left(\frac{dD}{dy}\right)_w - (U_y + \tau F_y) n_w\right] = (1+\chi)\left(\frac{\pi\tau}{2D_w}\right)^{1/2} J_w, \tag{14}$$

where $J_w = J_{inc} - J_{ref}$.

The terms in brackets in (14) define, respectively, the diffusion, migration, and convection particle flows at the surface.

The boundary condition for a completely absorbing surface is obtained from (14), where $\chi = 0$. For the reflecting surface (impermeable to the particles), the boundary condition assumes the form ($\chi = 1$)

$$J_w = 0 \quad \text{or} \quad \left(\frac{dDn}{dy}\right)_w = (U_y + \tau F_y)_w n_w.$$

It follows from (14) that the boundary condition $n_w = 0$ so popular in the literature is valid when $\varepsilon/(1 - \chi) \ll 1$.

NOTATION

v is the particle velocity; V, mean particle velocity; U, velocity of the medium; τ, time of dynamic particle relaxation; n, particle concentration; ℓ, characteristic dimension. Indices: w denotes the parameter at the wall.

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EFFECTS OF VISCOUS DISSIPATION AND JOULE HEAT ON HEAT TRANSFER NEAR A ROTATING DISK IN THE PRESENCE OF INTENSIVE SUCTION

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Heat transfer in the bounday layer of an electrically conducting incompressible liquid near a disk rotating in an axial magnetic field is investigated for the case of intensive, uniform suction. The thermal flux intensity near the disk surface is determined in relation to the magnetic field strength and the rotation speed of the disk with an allowance for the viscous and the Joule dissipation.

The characteristics of the hydrodynamic and the thermal boundary layers at a rotating unbounded permeable disk were calculated in [1, 2] by integrating the equations of motion and energy with averaged convective terms while neglecting the viscous dissipation. Heat transfer near a disk rotating in a conducting medium within an axial magnetic field was considered in the absence of suction [3] and in the case of strong suction [4], using a similar nondissipative approximation. We have considered the effect of viscous dissipation and of the Joule heat on heat transfer in the magnetohydrodynamic boundary layer at a permeable dielectric disk rotating in an electrically conducting incompressible, viscous medium. Let us assume that the difference between the temperature in the main flow and the disk temperature is relatively small [2]. We assume in accordance with [1, 2] that $w = w_0 - k$. Then, if condition $k \gg w_0$ is satisfied, we have the following for the thermal boundary layer at the disk:

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